

AP Calculus AB

Basic Integration

1) $f(x) = 4x^3$
 $\int 4x^3 dx = x^4 + C$

2) $f(x) = \sin(2x)$
 $\int \sin(2x) dx$
 $= -\frac{1}{2} \cos(2x) + C$

3) $\int 9 \sec^2 x dx$
 $[9 \tan x + C]$

4) $f(y) = \frac{-12}{y^3}$
 $\int -12y^{-3} dy$
 $[6y^{-2} + C]$

5) $f(x) = 2e^x$
 $\int 2e^x dx$
 $[2e^x + C]$

6) $\int (7x^{20} - 2x^5) dx$
 $\left[\frac{1}{3}x^{21} - \frac{1}{3}x^6 + C \right]$

7) $\int (9x^{-\frac{1}{2}} + 9x^{\frac{1}{2}}) dx$
 $\frac{9x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{9x^{\frac{3}{2}}}{\frac{3}{2}} + C$
 $[18x^{\frac{1}{2}} + 6x^{\frac{3}{2}} + C]$

8) $\int (4t^{-2} + 5t^6) dt$
 $[-4t^{-1} + \frac{5}{7}t^7 + C]$

9) $\int (3x+2)^2 dx$
 $\left[\frac{1}{9}(3x+2)^3 + C \right]$
check:
 $3(3x+2)^2 \cdot 3$

10) $\int (4x^{\frac{1}{3}} + 3x^{-\frac{1}{3}} + 9) dx$
 $\frac{4x^{\frac{4}{3}}}{\frac{4}{3}} + \frac{3x^{\frac{2}{3}}}{\frac{2}{3}} + 9x + C$
 $[3x^{\frac{4}{3}} + \frac{9}{2}x^{\frac{2}{3}} + 9x + C]$

11) $\int \frac{4x^5 + 10x^3}{x^2} dx$
 $\int (4x^5 + 10x^3)x^{-2} dx$
 $\int (4x^3 + 10x) dx$
 $[x^4 + 5x^2 + C]$

12) $\int (\sin 3y + \cos 7y) dy$
 $[-\frac{1}{3}\cos 3y + \frac{1}{7}\sin 7y + C]$

$$13) \int (-2\sec x \tan x - 5\sec^2 x) dx$$

$$\boxed{-2\sec x - 5\tan x + C}$$

$$14) \int (e^{2x} + 7x^{1/2}) dx$$

$$\boxed{\frac{1}{2}e^{2x} + \frac{14}{3}x^{3/2} + C}$$

$$15) \int \frac{6+u}{u} du$$

$$\int (6+u) \cdot u^{-1} du$$

$$\int (6u^{-1} + 1) du$$

$$\boxed{6\ln|u| + u + C}$$

$$16) \int \frac{16t^8+b}{t} dt$$

$$\int (16t^8 + b) \cdot t^{-1} dt$$

$$\int (16t^7 + b t^{-1}) dt$$

$$\boxed{\frac{8}{3}t^6 + b \ln|t| + C}$$

$$17) g(x) = \begin{cases} (x+4)^2, & x \leq -3 \\ 2x+7, & -3 < x < 4 \\ (5-x)^2, & x \geq 4 \end{cases}$$

$$\text{I. } \frac{x = -3}{g(-3) = 1} \quad \underline{\text{continuity}}$$

$$\text{II. } \lim_{x \rightarrow -3^-} g(x) = 1 \quad \left. \lim_{x \rightarrow -3} g(x) = 1 \right\}$$

$$\lim_{x \rightarrow -3^+} g(x) = 1$$

$$\text{III. } g(-3) = \lim_{x \rightarrow -3} g(x)$$

$$\frac{x=4}{g(4)=1}$$

$$\text{II. } \lim_{x \rightarrow 4^-} g(x) = 15 \neq \lim_{x \rightarrow 4^+} g(x) = 1$$

$g(x)$ is not cont @ $x=4$
which means $g(x)$ is not diff @ $x=4$.

$$\frac{d}{dx} [(x+4)^2] \Big|_{x=-3} = 2(x+4) \Big|_{x=-3} = 2$$

$$\frac{d}{dx} (2x+7) \Big|_{x=-3} = 2$$